Midterm Introduction to Logic

December 9th, 2013

- ☞ Part A consists of 3 exercises.
- ☞ Part B consists of 2 exercises.
- ☞ Philosophy students only need to do part A of the exam.
- ☞ Non-philosophy students should do both part A and part B.
- ☞ Only write your student number at the top of the exam. Also put your number at the top of any additional pages.
- ☞ Put the name of your group at the top of the exam.
- ☞ Use a blue or black pen (so no pencils, red pen or marker).

Good Luck!

Part A

A1: translation into propositional logic (24 points) Translate the following sentences to *propositional logic*. Atomic sentences are represented by uppercase letters. Do not forget to provide the translation key.

a. If you are small and wear a pointy hat you are a gnome, but if you are small and have a lot of hair on your feet you are a hobbit.

b. The human body has enough oxygen only if the heart pumps blood around and the lungs work.

c. I will not stop smoking if and only if it is not prohibited in all bars.

A2: translation into first-order logic (24 points) Translate the following sentences to *first-order logic*. Do not forget to provide the translation key.

a. Aarav and Yasmin both love Greg unless Greg loves Hans.

b. Only if John and Ananya are siblings, then Pranav and Shreya are not.

c. If Bill is not being nice and Mary is not having a good time, then Juhani takes him home or calls a taxi for him.

A3: formal proofs (32 points) Give formal proofs of the following inferences.

a. $(P \wedge Q \wedge R) \to S \vdash P \to (Q \to (R \to S))$

b. $\neg(P \vee Q) \vdash P \leftrightarrow Q$

c. $\neg(a = a \wedge P(a)) \vdash a = b \to \neg P(b)$

d. $\vdash P \to ((Q \vee \neg P) \to P)$

Part B

B1: Normal forms of propositional logic (20 points)

a. Provide a negation normal form (NNF) of: $\neg((P \vee Q) \wedge \neg\neg R)$

b. Provide a conjunction normal form (CNF) of: $\neg(P \wedge \neg Q) \rightarrow \neg(S \vee \neg R)$

B2: Set theory (20 points) Given the following three sets $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 6\}$ and $C = \{2, 3, 5, 6, 7\}$. For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

a. $5 \in A \cap B \cap C$

b. $(B \cup A) \setminus C \neq A \setminus B$

c. $6 \in C \cup \emptyset$

d. $A \cup B \subseteq B \cap C$

e. $(A \setminus B) \setminus C = \{1\}$

f. $(A \cup B) \setminus \emptyset \neq \emptyset$

g. $(A \cap B) \subseteq C$

h. $1 \notin (A \cup B) \cap (C \cup B)$

i. $C \setminus (C \cap B) \subseteq (A \cup B)$

j. $6 \notin (A \setminus B) \cap C$

Midterm

A1

a Translation key
S: You are small.
P: You wear a pointy hat.
G: You are a gnome.
H: You have a lot of hair on your feet
B: You are a hobbit

Translation: $((S \wedge P) \rightarrow G) \wedge ((S \wedge H) \rightarrow B)$

b Translation key
O: The human body has enough oxygen.
H: The heart pumps blood around.
L: The lungs work.

Translation: $O \rightarrow (H \wedge L)$

c Translation key
S: I will stop smoking.
P: Smoking is prohibited in all bars.

Translation: $\neg S \leftrightarrow \neg P$

Midterm

A2

a Translation key:

a: Aarav
y: Yashvi
g: Greg
h: Hans
$L(x,y)$: x loves y

Translation: $(L(a,g) \wedge L(y,g)) \vee L(g,h)$

b Translation key:

j: John
a: Anonya
p: Pranav
s: Shreya
$S(x,y)$: x and y are siblings

Translation: $\neg S(p,s) \rightarrow S(j,a)$

c Translation key:

b: Bill
m: Mary
j: Juhani
$N(x)$: x is nice
$G(x)$: x is having a good time
$H(x,y)$: x takes y home
$T(x,y)$: x calls a taxi for y

Translation: $(\neg N(b) \wedge \neg G(m)) \rightarrow (H(j,b) \vee T(j,b))$

Midterm

A3

a

1.	$(P \wedge Q \wedge R) \rightarrow S$	
2.	P	
3.	Q	
4.	R	
5.	$P \wedge Q \wedge R$	$\wedge$ Intro (2,3,4)
6.	S	$\rightarrow$ Elim (1,5)
7.	$R \rightarrow S$	$\rightarrow$ Intro (4-6)
8.	$Q \rightarrow (R \rightarrow S)$	$\rightarrow$ Intro (3-7)
9.	$P \rightarrow (Q \rightarrow (R \rightarrow S))$	$\rightarrow$ Intro (2-8)

b

1.	$\neg(P \vee Q)$	
2.	P	
3.	$P \vee Q$	$\vee$ Intro (2)
4.	$\perp$	$\perp$ Intro (1,3)
5.	Q	$\perp$ Elim (4)
6.	Q	
7.	$P \vee Q$	$\vee$ Intro (6)
8.	$\perp$	$\perp$ Intro (1,7)
9.	P	$\perp$ Elim (8)
10.	$P \leftrightarrow Q$	$\leftrightarrow$ Intro (2-5, 6-9)

Midterm

A3

c

1. $\neg(a=a \wedge P(a))$		
2. $a=b$		
3. $P(a)$		
4. $a=a$		$=$ Intro
5. $a=a \wedge P(a)$		$\wedge$ Intro (3, 4)
6. $\bot$		$\bot$ Intro (1, 5)
7. $\neg P(a)$		$\neg$ Intro (3–6)
8. $\neg P(b)$		$=$ Elim (2, 7)
9. $a=b \rightarrow \neg P(b)$		$\rightarrow$ Intro (2–8)

d

1. P		
2. $Q \vee \neg P$		
3. P		Reit (1)
4. $(Q \vee \neg P) \rightarrow P$		$\rightarrow$ Intro (2–3)
5. $P \rightarrow ((Q \vee \neg P) \rightarrow P)$		$\rightarrow$ Intro (1–4)

Midterm

B1

a $\neg((P \vee Q) \wedge \neg\neg R)$

$\Leftrightarrow$

$\neg((P \vee Q) \wedge R)$

$\Leftrightarrow$

$\neg(P \vee Q) \vee \neg R$

$\Leftrightarrow$

$(\neg P \wedge \neg Q) \vee \neg R$

b. $\neg(P \wedge \neg Q) \rightarrow \neg(S \vee \neg R)$

$\Leftrightarrow$

$\neg\neg(P \wedge \neg Q) \vee \neg(S \vee \neg R)$

$\Leftrightarrow$

$(P \wedge \neg Q) \vee \neg(S \vee \neg R)$

$\Leftrightarrow$

$(P \wedge \neg Q) \vee (\neg S \wedge \neg\neg R)$

$\Leftrightarrow$

$(P \wedge \neg Q) \vee (\neg S \wedge R)$

$\Leftrightarrow$

$(P \vee (\neg S \wedge R)) \wedge (\neg Q \vee (\neg S \wedge R))$

$\Leftrightarrow$

$(P \vee \neg S) \wedge (P \vee R) \wedge (\neg Q \vee (\neg S \wedge R))$

$\Leftrightarrow$

$(P \vee \neg S) \wedge (P \vee R) \wedge (\neg Q \vee \neg S) \wedge (\neg Q \vee R)$

Midterm

B 2

a false
b true
c true
d false
e true
f true
g true
h true
i false
j true